Differential Equations

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Differential Equations

Subtopics: Global or Absolute Minima and Maxima, Modelling Situations, Integration Technique - Harder Powers, Accumulation of Change, Total Amount, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Medium / Question Number: 4

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

Question 2

Qualification: AP Calculus AB

Areas: Differential Equations

Subtopics: Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Exponentials, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

Question 3

Qualification: AP Calculus AB

Areas: Differentiation, Differential Equations

Subtopics: Implicit Differentiation, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 6

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

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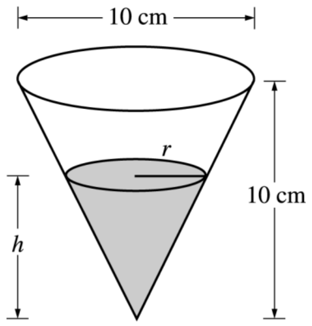
Question 4

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Rates of Change (Instantaneous), Modelling Situations, Related Rates

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Somewhat Challenging / Question Number: 5



5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

Question 5

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Local or Relative Minima and Maxima, Particular Solution of Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Very Hard / Question Number: 5

5. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

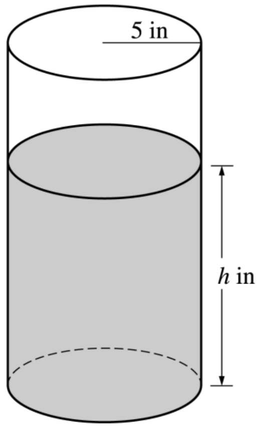
Question 6

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Integration

Subtopics: Related Rates, Particular Solution of Differential Equation, Integration Technique - Harder Powers, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Medium / Question Number: 5



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

Question 7

Qualification: AP Calculus AB

Areas: Differentiation, Differential Equations

Subtopics: Differentiation Technique – Chain Rule, Differentiation Technique – Product Rule, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2003-Form-B / Difficulty: Easy / Question Number: 6

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

Question 8

Qualification: AP Calculus AB

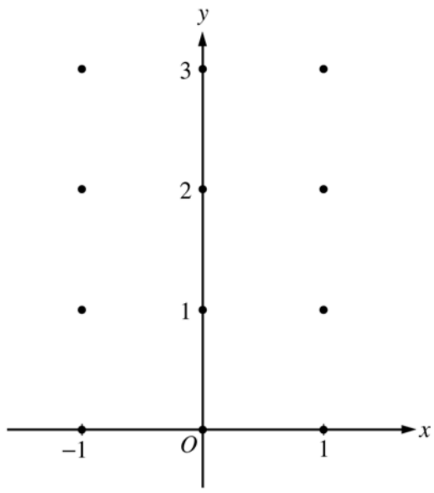
Areas: Differential Equations

Subtopics: Sketching Slope Field, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

Question 9

Qualification: AP Calculus AB

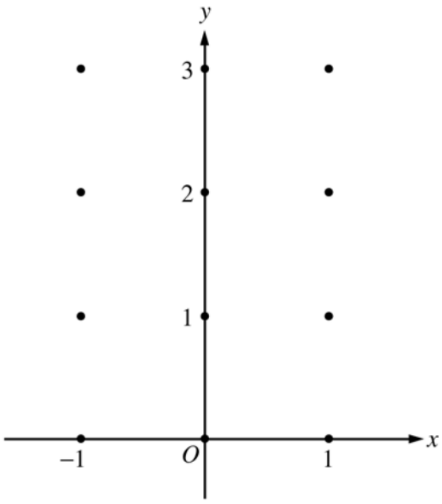
Areas: Differential Equations

Subtopics: Sketching Slope Field, Integration Technique - Harder Powers, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004-Form-B / Difficulty: Medium / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

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Question 10

Qualification: AP Calculus AB

Areas: Differential Equations

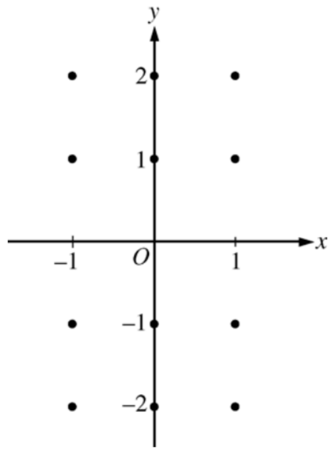
Subtopics: Sketching Slope Field, Tangents To Curves, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(**Note: Use the axes provided in the pink test booklet.**)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

Question 11

Qualification: AP Calculus AB

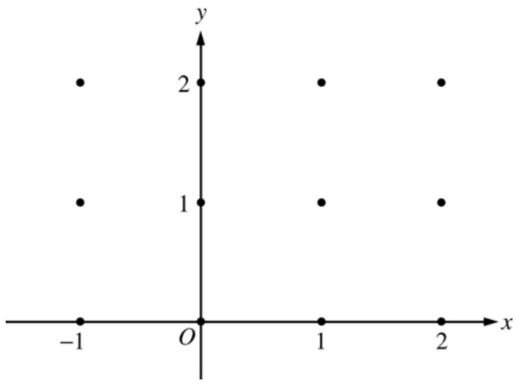
Areas: Differential Equations

Subtopics: Sketching Slope Field, Tangents To Curves, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2005-Form-B / Difficulty: Easy / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

Question 12

Qualification: AP Calculus AB

Areas: Differential Equations

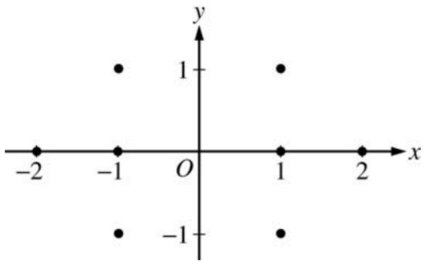
Subtopics: Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Sketching Slope Field, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2006 / Difficulty: Medium / Question Number: 5

5. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

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Question 13

Qualification: AP Calculus AB

Areas: Differential Equations

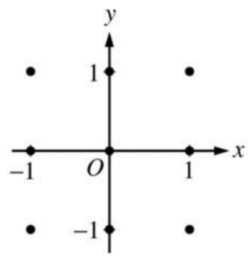
Subtopics: Sketching Slope Field, Integration Technique - Harder Powers, Integration Technique – Trigonometry, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation

Paper: Part B-Non-Calc / Series: 2006-Form-B / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

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Question 14

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Differentiation Technique – Chain Rule, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 4

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

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Question 15

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

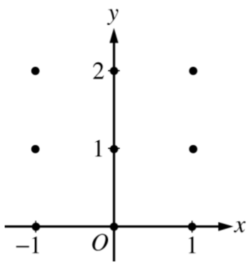
Subtopics: Sketching Slope Field, Concavity, Initial Conditions in Differential Equation, Local or Relative Minima and Maxima, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

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Question 16

Qualification: AP Calculus AB

Areas: Differential Equations, Limits and Continuity

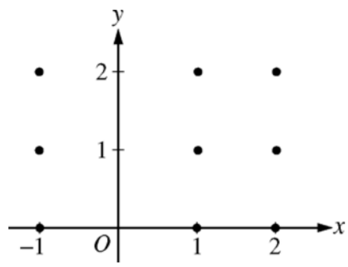
Subtopics: Sketching Slope Field, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Standard Functions, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

   (Note: Use the axes provided in the exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

   (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

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Question 17

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Easy / Question Number: 6

6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

   (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

   (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

   (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

Question 18

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Medium / Question Number: 5

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

   (b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

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Question 19

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differential Equations

Subtopics: Derivative Graphs, Concavity, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Initial Conditions in Differential Equation, Particular Solution of Differential Equation

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Somewhat Challenging / Question Number: 5

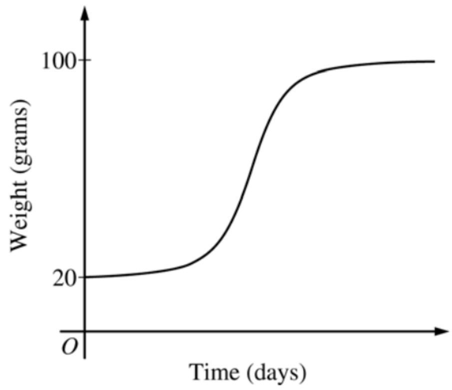
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Question 20

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

   (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

   (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

Question 21

Qualification: AP Calculus AB

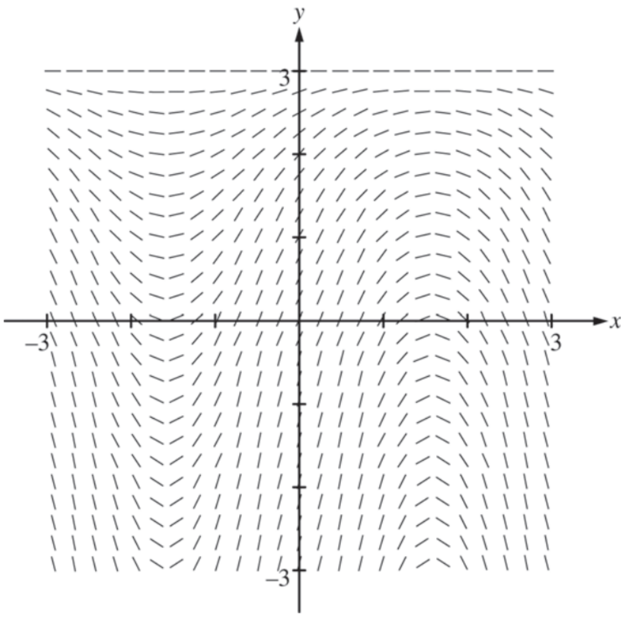
Areas: Applications of Differentiation, Differential Equations

Subtopics: Sketching Slope Field, Tangents To Curves, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

Question 22

Qualification: AP Calculus AB

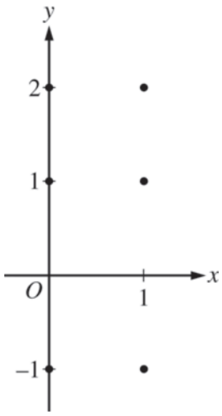
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Concavity, Local or Relative Minima and Maxima, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Medium / Question Number: 4

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

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Question 23

Qualification: AP Calculus AB

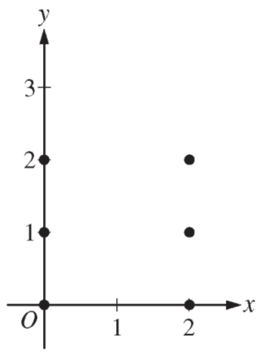
Areas: Applications of Differentiation, Differential Equations

Subtopics: Sketching Slope Field, Tangents To Curves, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Easy / Question Number: 4

4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

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Question 24

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Somewhat Challenging / Question Number: 4

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

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Question 25

Qualification: AP Calculus AB

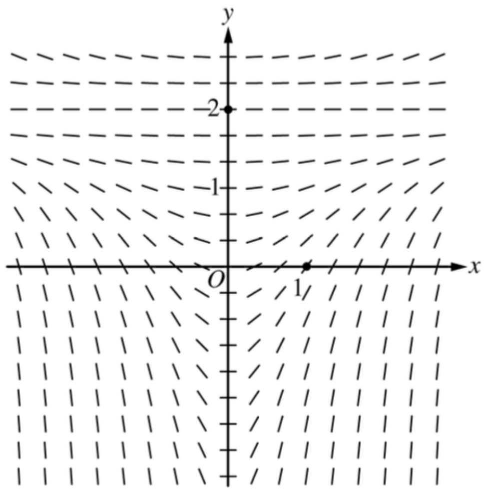
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Tangents To Curves, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Easy / Question Number: 6

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

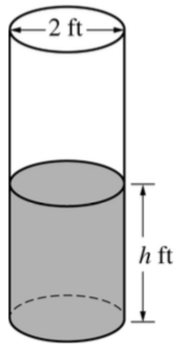
Question 26

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Rates of Change (Instantaneous), Increasing/Decreasing , Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Integration Technique - Harder Powers, Related Rates

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Somewhat Challenging / Question Number: 4



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

(b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

(c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

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Question 27

Qualification: AP Calculus AB

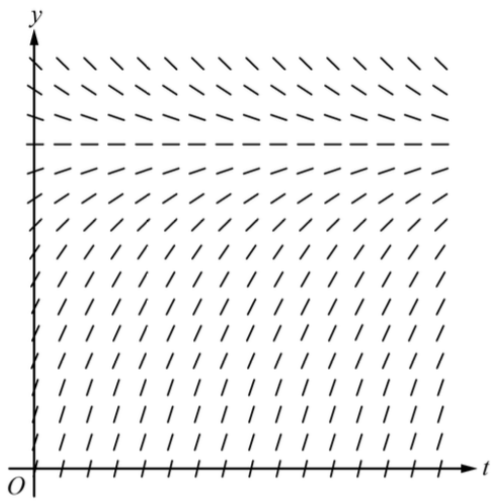
Areas: Applications of Differentiation, Differentiation, Differential Equations

Subtopics: Sketching Slope Field, Interpreting Meaning in Applied Contexts, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Increasing/Decreasing , Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Medium / Question Number: 6

6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t + 2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

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Question 28

Qualification: AP Calculus AB

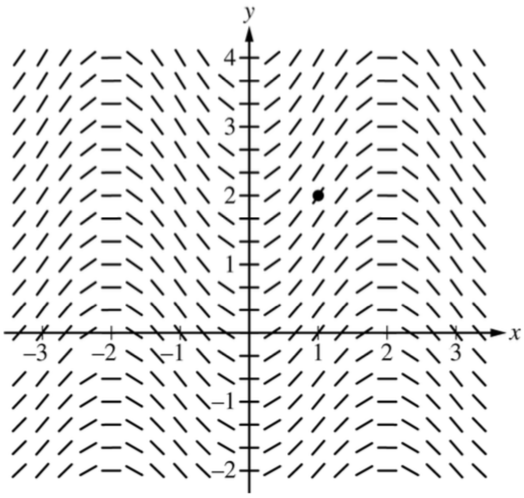
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Tangents To Curves, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Trigonometry, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

Question 29

Qualification: AP Calculus AB

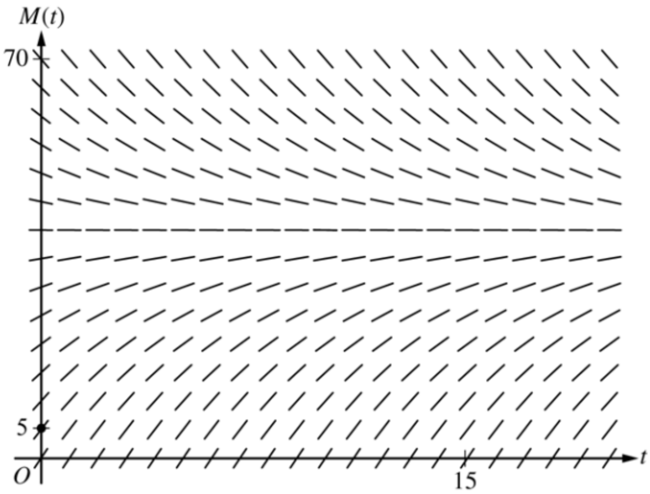
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Tangents To Curves, Concavity, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: / Question Number: 3

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

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Question 30

Qualification: AP Calculus AB

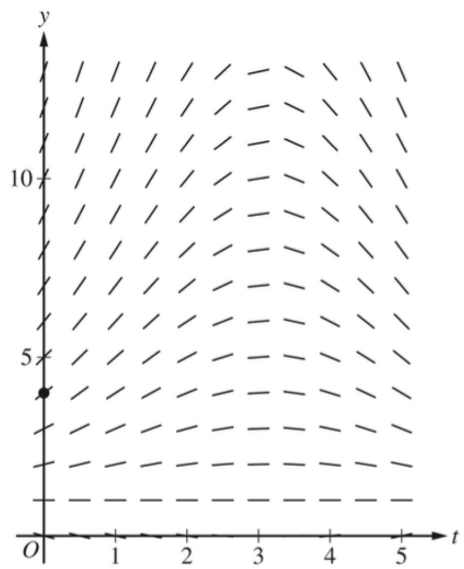
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Local or Relative Minima and Maxima, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Trigonometry

Paper: Part B-Non-Calc / **Series:** 2024 / **Difficulty:** Medium / **Question Number:** 3

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

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